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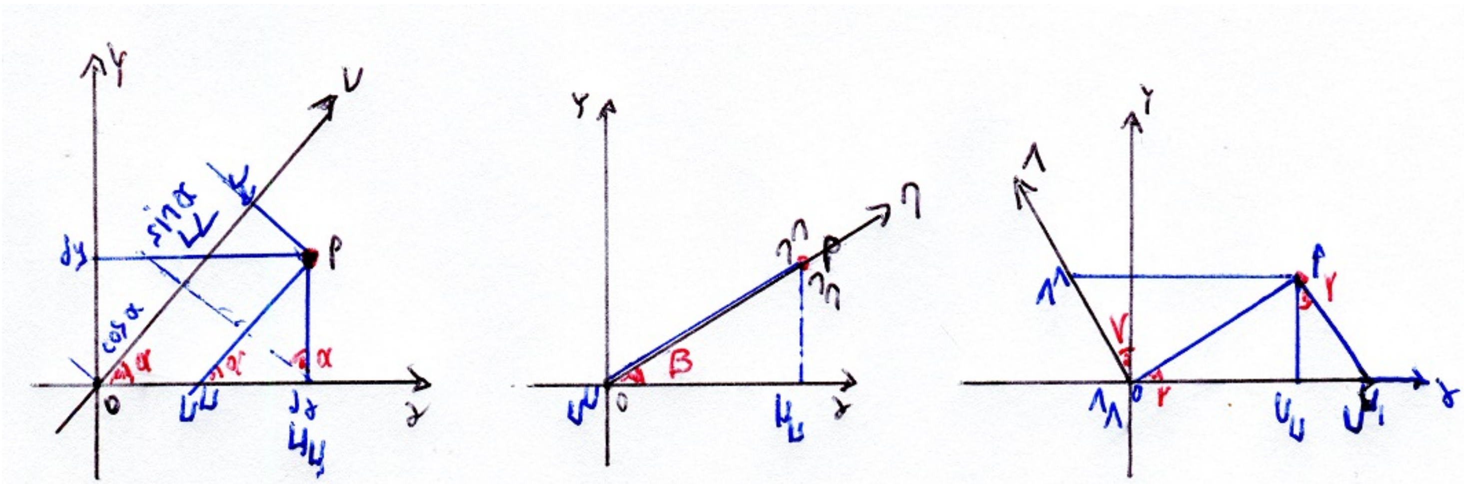
UNDER CONSTRUCTION



y axis rotation to right with $\mathbf{P} = \left(\mathbf{x}^{+}\ ,\ \mathbf{y}^{+}\right)$ .... y axis rotation up to the straight line 0P .... y axis rotation up to the $\overline{\mathbf{0P} + \frac{\pi}{2}}$ (in second Quadrant)

$\mathbf{0}$ = the Observer ; $\mathbf{P}$ = the Object

* below the trigonometric functions raised to **-n** are not to be understood as inverse functions but as $\mathbf{f}^{-n} = \dfrac{\mathbf{1}}{\mathbf{f}^{n}}$

* in the equations below **x** = ***d*x** , **y** = ***d*y**

first case: $\mathbf{P}\left(\mathbf{x}^{+}\ ,\ \mathbf{y}^{+}\right)$ *and Y axis rotations up to* $\overline{\mathbf{0P}}$ *and* $\overline{\mathbf{0P} + \frac{\pi}{2}}$ *:*

$$\begin{cases} \mu^{\mu} = dx - \dfrac{dy}{\tan\alpha} \\ \nu^{\nu} = \dfrac{dy}{\sin\alpha} \end{cases} \quad \text{....................} \quad H = \begin{bmatrix} 1 & \tan^{-1}\alpha \\ 0 & \sin^{-1}\alpha \end{bmatrix}$$

geometric transformation matrix to get contravariant components by cartesian coordinates

$$\begin{cases} \mu_{\mu} = dx \\ \nu_{\nu} = dx\cos\alpha + dy\sin\alpha \end{cases} \quad \text{....................} \quad M = \begin{bmatrix} 1 & 0 \\ \cos\alpha & \sin\alpha \end{bmatrix}$$

geometric transformation matrix to get covariant components by cartesian coordinates

$$\begin{cases} \mu_{\mu} = \mu^{\mu} + \nu^{\nu}\cos\alpha \\ \nu_{\nu} = \nu^{\nu} + \mu^{\mu}\cos\alpha \end{cases} \quad \text{....................} \quad g_{\mu\nu}\begin{bmatrix} \mu^{\mu} \\ \nu^{\nu} \end{bmatrix} = \begin{bmatrix} \mu_{\mu} \\ \nu_{\nu} \end{bmatrix} \quad \text{....................} \quad g_{\mu\nu} = \begin{bmatrix} 1 & \cos\alpha \\ \cos\alpha & 1 \end{bmatrix}$$

covariant transformation matrix to get covariant components by contravariant components

$$\begin{cases}\mu^{\mu} = \sin^{-2}\alpha\left(\mu_{\mu} - \nu_{\nu}\cos\alpha\right)\\ \nu^{\nu} = \sin^{-2}\alpha\left(\nu_{\nu} - \mu_{\mu}\cos\alpha\right)\end{cases}$$ .... $$g^{\mu\nu}\begin{bmatrix}\mu_{\mu}\\ \nu_{\nu}\end{bmatrix} = \begin{bmatrix}\mu^{\mu}\\ \nu^{\nu}\end{bmatrix}$$ .... $$g^{\mu\nu} = \begin{bmatrix}\sin^{-2}\alpha & \frac{-\cos\alpha}{\sin^{2}\alpha}\\ \frac{-\cos\alpha}{\sin^{2}\alpha} & \sin^{-2}\alpha\end{bmatrix}$$ *contravariant transformation matrix to get contravariant components by covariant components*

$$HM^{-1} = g^{\mu\nu}g_{\mu\nu} = I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

(Cartesian coordinates must be transposed, so the cartesian matrix to get covariant vector, must be transposed)

second case: y axis rotation up to the straight line $\overline{0P}$

$$\begin{cases}\mu^{\mu} = 0\\ \eta^{\eta} = \frac{dx}{\cos\beta} = \frac{dy}{\sin\beta}\end{cases} \qquad A = \begin{bmatrix}0 & 0\\ \cos^{-1}\beta & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & \sin^{-1}\beta\end{bmatrix}$$

$$\begin{cases}\mu_{\mu} = dx\\ \eta_{\eta} = \frac{dx}{\cos\beta} = \frac{dy}{\sin\beta}\end{cases} \qquad B = \begin{bmatrix}1 & 0\\ \cos^{-1}\beta & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & \sin^{-1}\beta\end{bmatrix}$$

$$\begin{cases}\mu_{\mu} = \eta^{\eta}\cos\beta\\ \eta_{\eta} = \eta^{\eta}\end{cases} \qquad g_{\mu\eta} = \begin{bmatrix}0 & \cos\beta\\ 0 & 1\end{bmatrix}$$

$$\begin{cases}\mu^{\mu} = 0\\ \eta^{\eta} = \eta_{\eta} = \frac{\mu_{\mu}}{\cos\beta}\end{cases} \qquad g^{\mu\eta} = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ \cos^{-1}\beta & 0\end{bmatrix}$$

third case: y axis rotation up to the straight line $\overline{0P + \frac{\pi}{2}}$ *(in second Quadrant)*

$$\begin{cases} \mu_\mu = x \\ \Lambda_\Lambda = 0 \end{cases} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{cases} \mu^\mu = x + y \tan\gamma \\ \Lambda^\Lambda = \dfrac{y}{\cos\gamma} \end{cases} \qquad D = \begin{bmatrix} 1 & \tan\gamma \\ 0 & \cos^{-1}\gamma \end{bmatrix}$$

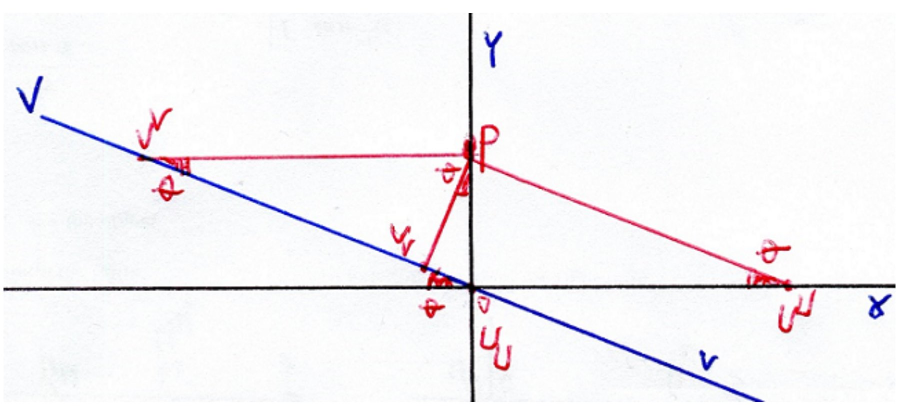
$$\begin{cases} \mu_\mu = \mu^\mu - \Lambda^\Lambda \sin\gamma \\ \Lambda_\Lambda = 0 \end{cases} \qquad g_{\mu\Lambda} = \begin{bmatrix} 1 & -\sin\gamma \\ 0 & 0 \end{bmatrix}$$

$$\begin{cases} \mu^\mu = \dfrac{\Lambda^\Lambda}{\sin\gamma} = \dfrac{\mu^\mu - \mu_\mu}{\sin^2\gamma} = \dfrac{-\mu_\mu}{\sin^2\gamma - 1} = \dfrac{\mu_\mu}{\cos^2\gamma} \\ \Lambda^\Lambda = \mu^\mu \sin\gamma = \dfrac{\mu_\mu \sin\gamma}{\cos^2\gamma} \end{cases} \qquad g^{\mu\Lambda} = \begin{bmatrix} \dfrac{1}{\cos^2\gamma} & 0 \\ \dfrac{\tan\gamma}{\cos\gamma} & 0 \end{bmatrix}$$

if the rotation axis is up to the $\overline{0P + \frac{\pi}{2}}$ *or up to the straight line* $\overline{0P}$ *, the metric tensor is not invariant:*

$$AB^{-1} \neq g^{\mu\eta} g_{\mu\eta} \neq I \neq AB^{-1} \;,\; DC^{-1} \neq g^{\mu\Lambda} g_{\mu\Lambda} \neq I \neq DC^{-1}$$

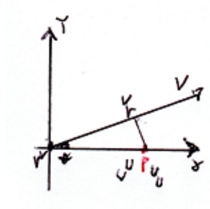
Also if $\overline{0P}$ is up to the ***y*** axis, then it cannot keep its distance invariant



$$\begin{cases} \mu^\mu = \dfrac{y}{\tan\theta} \\ \nu^\nu = \dfrac{y}{\sin\theta} \end{cases} \qquad \Psi = \begin{bmatrix} 0 & \dfrac{1}{\tan\theta} \\ 0 & \dfrac{1}{\sin\theta} \end{bmatrix}$$

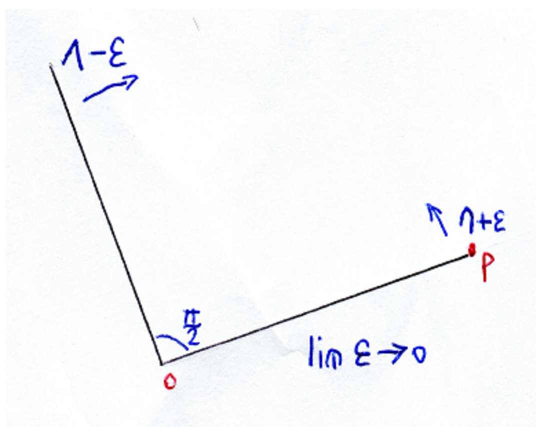
$$\begin{cases} \mu_\mu = 0 \\ \nu_\nu = y \sin\theta \end{cases} \qquad \xi = \begin{bmatrix} 0 & 0 \\ 0 & \sin\theta \end{bmatrix}$$

$$\Psi \xi^T = \begin{bmatrix} 0 & \cos\theta \\ 0 & 1 \end{bmatrix} \neq I$$

Also assuming the case of P on x axis  $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}1 & \cos\theta\\0 & 0\end{bmatrix}=\begin{bmatrix}1 & \cos\theta\\0 & 0\end{bmatrix}$

fourth case: the Y rotations are not exactly coincident with the straight lines $\overline{\mathbf{0P}}$ *and* $\overline{\mathbf{0P}+\frac{\pi}{2}}$ *:*

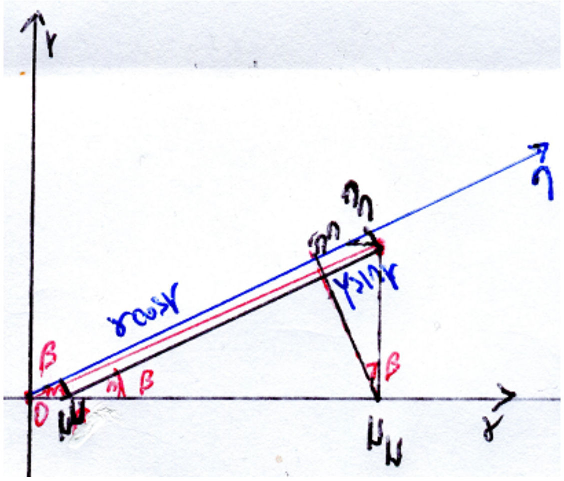
$\left[\mathbf{0P}+\frac{\pi}{2}-\varepsilon \quad , \quad \mathbf{0P}+\varepsilon\right] \quad | \lim \varepsilon \to 0$



the case of the quasi-straight line 0P is:

$$\begin{cases}\mu^{\mu}=x-y\cot\beta\\ \eta^{\eta}=\dfrac{y}{\sin\beta}\end{cases} \quad \text{..........} \quad M=\begin{bmatrix}1 & -\cot\beta\\0 & \dfrac{1}{\sin\beta}\end{bmatrix}$$

$$\begin{cases}\mu_{\mu}=x\\ \eta_{\eta}=x\cos\beta+y\sin\beta\end{cases} \quad \text{..........} \quad N=\begin{bmatrix}1 & 0\\ \cos\beta & \sin\beta\end{bmatrix}$$

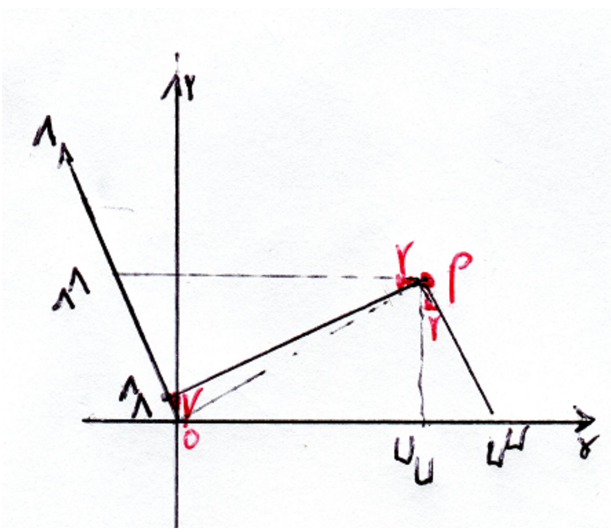


$$\begin{cases} \mu_\mu = \mu^\mu + \eta^\eta \cos\beta \\ \eta_\eta = \mu^\mu \cos\beta + \eta^\eta \end{cases} \quad \ldots\ldots\ldots\ldots \quad g_{\mu\eta} = \begin{bmatrix} 1 & \cos\beta \\ \cos\beta & 1 \end{bmatrix}$$

$$\begin{cases} \mu^\mu = \mu_\mu - \eta^\eta \cos\beta = \dfrac{\mu_\mu - \eta_\eta \cos\beta}{\sin^2\beta} \\ \eta^\eta = \eta_\eta - \mu^\mu \cos\beta = \dfrac{\eta_\eta - \mu_\mu \cos\beta}{\sin^2\beta} \end{cases} \quad \ldots\ldots\ldots\ldots \quad g^{\mu\eta} = \begin{bmatrix} \dfrac{1}{\sin^2\beta} & \dfrac{-\cos\beta}{\sin^2\beta} \\ \dfrac{-\cos\beta}{\sin^2\beta} & \dfrac{1}{\sin^2\beta} \end{bmatrix}$$

$$MN^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = g^{\mu\eta} g_{\mu\eta} = I$$

the case of the quasi-$\overline{0P + \frac{\pi}{2}}$ *is (noting that it is in the second quadrant) :*



$$\begin{cases} \mu_\mu = x \\ \Lambda_\Lambda = \dfrac{y}{\cos\gamma} - x\sin\gamma - y\dfrac{\sin^2\gamma}{\cos\gamma} = -x\sin\gamma + y\left(\dfrac{1-\sin^2\gamma}{\cos\gamma}\right) = -x\sin\gamma + y\cos\gamma \end{cases} \quad \ldots\ldots\ldots\ldots \quad E = \begin{bmatrix} 1 & 0 \\ -\sin\gamma & \cos\gamma \end{bmatrix}$$

$$\begin{cases} \mu^\mu = x + y\tan\gamma \\ \Lambda^\Lambda = \dfrac{y}{\cos\gamma} \end{cases} \quad \ldots\ldots\ldots\ldots \quad F = \begin{bmatrix} 1 & \tan\gamma \\ 0 & \cos^{-1}\gamma \end{bmatrix}$$

$$FE^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\begin{cases} \mu_\mu = \mu^\mu - \Lambda^\Lambda \sin\gamma \\ \Lambda_\Lambda = -\mu^\mu \sin\gamma + \Lambda^\Lambda \end{cases} \qquad g_{\mu\Lambda} = \begin{bmatrix} 1 & -\sin\gamma \\ -\sin\gamma & 1 \end{bmatrix}$$

$$\mu^\mu = \mu_\mu + \Lambda^\Lambda \sin\gamma = \mu_\mu + \left(\Lambda_\Lambda + \mu^\mu \sin\gamma\right)\sin\gamma = \mu_\mu + \Lambda_\Lambda \sin\gamma + \mu^\mu \sin^2\gamma =$$

$$= \mu_\mu + \Lambda_\Lambda \sin\gamma + \mu^\mu - \mu^\mu \cos^2\gamma = \frac{\mu_\mu + \Lambda_\Lambda \sin\gamma}{\cos^2\gamma}$$

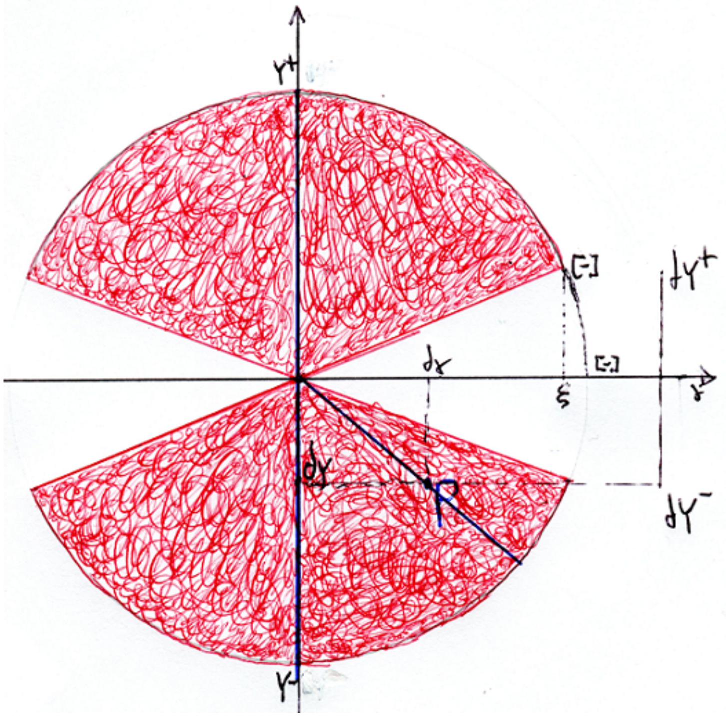
$$\Lambda^\Lambda = \Lambda_\Lambda + \mu^\mu \sin\gamma = \Lambda_\Lambda + \frac{\mu_\mu \sin\gamma + \Lambda_\Lambda \sin^2\gamma}{\cos^2\gamma} = \frac{\mu_\mu \sin\gamma + \Lambda_\Lambda}{\cos^2\gamma}$$

$$\begin{cases} \mu^\mu = \dfrac{\mu_\mu + \Lambda_\Lambda \sin\gamma}{\cos^2\gamma} \\ \Lambda^\Lambda = \dfrac{\mu_\mu \sin\gamma + \Lambda_\Lambda}{\cos^2\gamma} \end{cases} \qquad g^{\mu\Lambda} = \begin{bmatrix} \dfrac{1}{\cos^2\gamma} & \dfrac{\sin\gamma}{\cos^2\gamma} \\ \dfrac{\sin\gamma}{\cos^2\gamma} & \dfrac{1}{\cos^2\gamma} \end{bmatrix}$$

$$g^{\mu\Lambda} g_{\mu\Lambda} = \begin{bmatrix} \dfrac{1}{\cos^2\gamma} & \dfrac{\sin\gamma}{\cos^2\gamma} \\ \dfrac{\sin\gamma}{\cos^2\gamma} & \dfrac{1}{\cos^2\gamma} \end{bmatrix} \begin{bmatrix} 1 & -\sin\gamma \\ -\sin\gamma & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

the distance 0 to P is invariant

Thus the Left ***L*** and Right ***R*** Limits of the rotation of the **Y** Axis are $\overline{\mathbf{0}\ \left(\left[\mathbf{x} \to \pm\infty \mp \boldsymbol{\varepsilon}\right],\left[\mathbf{y} \to d\mathbf{y}^{+-}\right]\right)}$, this Rotation must be within the Boundaries $\boldsymbol{L}, \boldsymbol{R}$, id est the 2D Rotation Space in the metric tensor can exist within the closed and disconnected set $(\boldsymbol{L}, \boldsymbol{R})$ only.



The area of the Metric Tensor 2D Rotation Space (the 'Clepsydra') is:

$$\varsigma = \pi \cdot \Xi^2 - 3\Xi - 2d\mathbf{y}\xi - 4\int_{\xi}^{\Xi}\sqrt{1+\left(\frac{-\mathbf{x}}{\sqrt{\Xi^2-\mathbf{x}^2}}\right)^2}\,d\mathbf{x}$$

where $\Xi = \overline{\mathbf{0}\ \left(\left[\mathbf{x} \to \pm\infty \mp \boldsymbol{\varepsilon}\right],\left[\mathbf{y} \to d\mathbf{y}^{+-}\right]\right)}$

and where $\xi = \Xi \cos\gamma$

TO BE CONTINUED ....