Transcendence of e^e

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The method here used to prove the transcendence of ***exp(e)*** is very similar to the one already used to demonstrate the transcendence of *exp(algebraic number)* (1).

$I(t) = \int_0^t e^{t-x} fx \ \ dx$ is a holomorphic function, differentiable over the complex numbers (2) in every point of its domain, and its integrand is a polynomial of a certain degree m , in which the exponent of the denominator slides all domain values **[0,t]**

Its input variables are conjugates indexed by a table r_s made of **r=rows** and **s=columns** compatible with the modular form (3)

We exploit all the properties of complex conjugation including those about the odd-degree polynomials, in accordance with the complex conjugate root theorem.

We assume $t = \cos(k\theta) - i \sin(k\theta)$, i.e. a conjugate in polar form, with *k* integer.
Each of the infinite values that θ_ϑ can assume, is closely related to π or its multiple or its fraction. For this reason we must assume that the value of the multiple or the value of the fraction of π can also be non-algebraic. Then e^θ returns algebraic values, (4)

Integrating by parts and assuming $y = t - x \quad ; \quad d(-y) = dx$, we get:

$-e^{-y} f(t-y) + \int_0^t e^{-y} f'(t-y) \ \ dy$ then $\left[-e^{-y} f^j(t-y)\right]^{t-0}$ then

$$I(t) = -e^{-t} \sum_{j=0}^{m} f^j(0) \quad - \sum_{j=0}^{m} f^j(t)$$

with m = degree of f and f^j = j-th derivative of f.

Let a symmetric polynomial $a_0 + a_1 e^{-i\theta} + a_2 e^{-2i\theta} + \cdots \ldots \ldots + a_n e^{-ni\theta} =$
$f_h = \prod_{q=1}^{n} (e^{-ik\theta} - w x_q)^p \ / (e^{-ik\theta} - w x_h)$

with degree $np - 1$, with $h \in [q]$, $1 \le q \le n$, $0 \le k \le n$, and p is a Prime sufficiently large.
x_q are distinct algebraic complex conjugate linearly independent. Appropriate coefficients a_k and w make x_q root of f
This polynomial is never negative.

Then we use next polynomial with $a_0, \ldots \ldots, a_n$ integers non-zero , to verify the possibility of existence of an algebraic result $J = a_0 I(0) + a_1 I(1) + \ldots \ldots \ldots + a_n I(n) =$
$\sum_{k=0}^{n} a_k I(k) = \sum_{k=0}^{n} a_k \left(\sum_{j=0}^{m} -e^{eik\theta} f^j(0) - \sum_{j=0}^{m} f^j(e^{-ik\theta}) \right) = -\sum_{j=0}^{m} \sum_{k=0}^{n} a_k f^j(e^{-ik\theta})$ (derivative of $f(0) = 1$ = null).

Considering that $f^{p-1}(0) = (p-1)!$ by $p-1$ derivations, and assuming $X = e^{-ik\theta} - x_q$, we can extract from f , by $p-1$ derivations, the polynomial $X^{(n-1)p}((p-1)! + (n-1)p!)$, then , the minimal polynomial $(p-1)!\, X^{(n-1)p}$ is divisible by $(p-1)!$ and it follows that $|J| > (p-1)!$

Considering that ***a*** , ***n*** and ***t*** have not infinite values , so $\prod_{h=1}^{n} |J_h| > I(t)$ is defined in a bounded set, therefore there must be a number greater than ***J*** . This number could be an arbitrary c^p . So we have :

$$|J| > (p-1)! > c^p > |J|$$

(1)
let us avoid the immediate and trivial demonstration : $e^e = e^{n+1} = e^{algebraic\ number}$

(2)
The use of complex numbers ensures that every non-constant polynomial has a root, since the Fundamental Theorem of Algebra states that every non-constant polynomial with coefficients in ***C*** , has zeros in ***C*** , false in ***R*** (typical instance: x^2 has zero in complex numbers only).

(3)
a redundance of complex numbers in upper half-plane in which each point of each of the two axes is intersected by a two-dimensional table composed of complex numbers, id est an object in four spatial dimensions, which returns only positive values, not drawable on graph.

(4)
So, we get *exp(algebraic number)* again!