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Proof of the Transcendence of $\mathbf{e^e}$

$$I(t) = \int_0^t e^{e^{t-x}} f(x)\ dx$$

$$\begin{cases} x > t & \rightarrow \quad 0 < \dfrac{e^{e^t}}{e^{e^x}} < 1 \\ t\backslash\{\infty\} \ > x & \rightarrow \quad 1 < \dfrac{e^{e^t}}{e^{e^x}} < M \end{cases}$$

Thus $\int e^{et-ex} = Q \in (0,M)$ ***id est*** $\dfrac{e^{e^t}}{e^{e^x}}$ ***is a not well defined number of which we do not know the numeric set***

$Q = \dfrac{e^{e^t}}{e^{e^x}}$ ***is an unknown constant, but in any case it is included in to the range (0,M).***

Then m ***is the degree of*** $f(x)$***. So, we integrate*** $I(t)$ ***by parts*** m ***times:***

$$Q\left\{\left[\ xf(x)\ \right]_0^t - \int_0^t f^{(j)}(x)\ dx\right\} \ldots\ldots\ldots =$$

$$= Q\left[xf^{(j)}(x)\right]_0^t = Qt\sum_{j=0}^{m} f^{(j)}(t)$$

where m ***is the degree of the integrand and*** $f^{(j)}$ ***is the jth derivative of*** f ***,***

its input is an arbitrary set of distinct conjugate complex numbers taken over the line joining [0,t]
The use of complex numbers ensures that this non-constant polynomial has roots (fundamental theorem of Algebra), and the complex conjugation give us real roots if m *is an odd degree (complex conjugate root theorem).*

$$t = \left[\alpha_{1_1}, \ldots, \alpha_{1_n}, \ldots, \alpha_{2_1}, \ldots, \alpha_{2_n}, \ldots, \alpha_{n_1}, \ldots, \alpha_{n_n}\right]$$ *is a complete set of conjugates.*

α_{k_h} *are distinct algebraic numbers linearly independent over the rationals.*

$$\begin{matrix} \alpha_{1_1} & \alpha_{1_2} & \alpha_{1_3} & \alpha_{1_4} & \cdot\cdot & \cdot\cdot \\ \alpha_{2_1} & \alpha_{2_2} & \alpha_{2_3} & \alpha_{2_4} & \cdot\cdot & \cdot\cdot \\ \alpha_{3_1} & \alpha_{3_2} & \alpha_{3_3} & \alpha_{3_4} & \cdot\cdot & \cdot\cdot \\ \alpha_{4_1} & \alpha_{4_2} & \alpha_{4_3} & \alpha_{4_4} & \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \end{matrix}$$

the conjugate plane 1 intersects the conjugate plane 1 then the plane 2 intersects the plane 1 and so on...
the intersections of the points generate overlaps and misalignaments beetween 'h' and 'k' planes

n^2 *is the cardinality of the* $t = \alpha_{k_h}$ *series and the number of all permutations of its values is* $\prod_{t=1}^{n^2} t$

$$n \in \mathbb{N} \backslash \{\infty\} \qquad 1 \leq i,y,k,h \leq n$$

λ *is the positive integer leading coefficient in the minimal polynomial defining* t

$$f_{i_y}(t) = \frac{\lambda^{nnp} \left\{ \prod_{\substack{k= \\ h=}[1,n]}^{n} \left(t - \alpha_{k_h}\right) \right\}^p}{\left(t - \alpha_{i_y}\right)}$$ *, where* $p \backslash \{\infty\}$ *is a very large Prime*

β_{k_h} *are non-zero integers. Then by our hypothesis of* $\mathbf{e}^{\mathbf{e}}$ *Algebraic number, have*

$$\beta_{1_1} e^{e^{\alpha_{1_1}}} + ... + \beta_{2_1} e^{e^{\alpha_{2_1}}} + ... + \beta_{2_n} e^{e^{\alpha_{2_n}}} + ... + \beta_{n_1} e^{e^{\alpha_{n_1}}} + ... + \beta_{n_n} e^{e^{\alpha_{n_n}}} = 0$$

$$J_{i_y} = Q \sum_{j=0}^{nnp-1} \sum_{\substack{k= \\ h=}[1,n]}^{n} \alpha_{k_h} \beta_{k_h} f^{(j)}(\alpha_{k_h})$$

then we use

where f ***is a symmetric polynomial that derivatived by substitution from*** **zero** ***to*** **nnp-2** ***times returns*** **value = zero** ***, but if derivatived by substitution*** **nnp-1** ***times returns an integer that is divisible by*** $(p-1)!$ ***but not by*** $p!$

$$J_{n_n} = n^2 J_{i_y}$$

then, we use the lemma on transcendence given by the Liouville theorem :

$$\left| a - \frac{b}{c} \right| > \frac{K}{c^n}$$

, where, by our hypothesis, **a** ***is an irrational and algebraic number, and*** **b** ***and*** **c** ***are integers , and*** K ***is a Lipschitz constant sufficiently large***

$$\frac{b}{c} = (p-1)! = \frac{C(p-1)! = b}{C = c}$$

where C ***is an integer near to*** $\to \infty$

e^e ***is an irrational number, root of the minimal polynomial*** Q

we use the properties of the Lipschitz inequality:

$$K\left|e^{e} - (p-1)!\right| \;>\; \left|Q - f_{i_y}\right|$$

$$\left|e^{e} - (p-1)!\right| \;>\; \frac{1}{K}\left|Q - f_{i_y}\right| \;>\; \frac{K}{C^{n}}$$