Author: Renzo Diomedi

Assuming

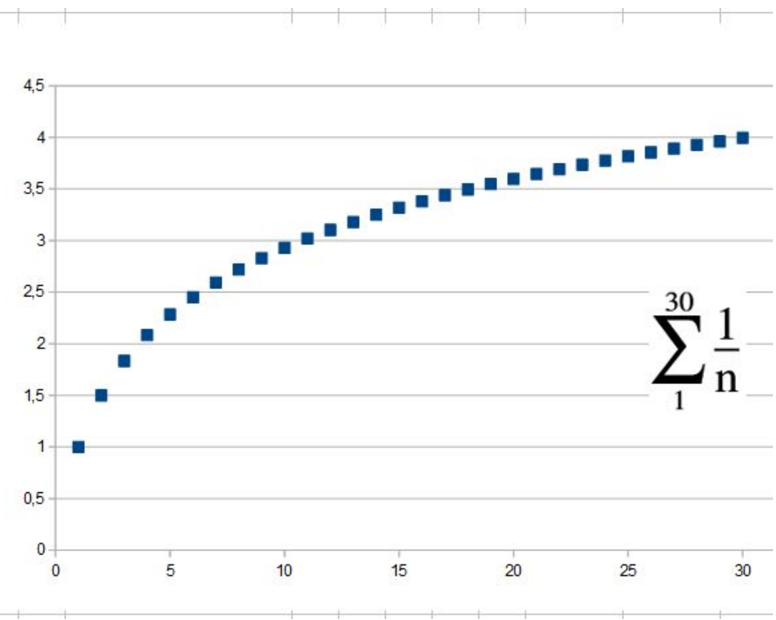
$$\begin{cases} \zeta(1) \\ \varepsilon \to 0 \\ n = +\infty - \varepsilon \end{cases}$$

$$\gamma = \sum_{1}^{n} \frac{1}{k} - \int_{1}^{n} \frac{1}{k}\, dk = \sum_{1}^{n} \frac{1}{k} - \left[\log k\right]_{1}^{n} = \sum_{1}^{n} \frac{1}{k} - \log n =$$

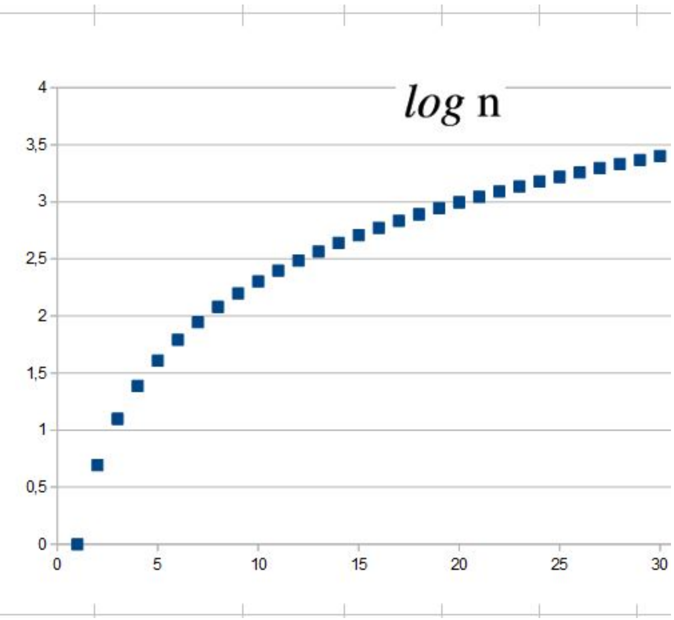
$$= H_n - \log n \sim 0.5772156649................$$

we arbitrarily assume $k \in \mathbb{N}$***. thus the Euler+Mascheroni equation is the partial harmonic series minus the primitive of its function***

1	1
2	1,5
3	1,8333333333
4	2,0833333333
5	2,2833333333
6	2,45
7	2,5928571429
8	2,7178571429
9	2,828968254
10	2,928968254
11	3,0198773449
12	3,1032106782
13	3,1801337551
14	3,2515623266
15	3,3182289932
16	3,3807289932
17	3,4395525226
18	3,4951080782
19	3,5477396571
20	3,5977396571
21	3,6453587048
22	3,6908132502
23	3,7342915111
24	3,7759581778
25	3,8159581778
26	3,8544197162
27	3,8914567533
28	3,927171039
29	3,9616537976
30	3,9949871309



1	0	
2	0,6931471368	....
3	1,0986122193	....
4	1,3862942736	....
5	1,6094378109	....
6	1,7917593562	....
7	1,9459100263	....
8	2,0794414105	....
9	2,1972244387	....
10	2,3025849477	....
11	2,3978951215	....
12	2,484906493	....
13	2,5649491956	....
14	2,6390571631	....
15	2,7080500302	....
16	2,7725885473	....
17	2,8332131653	....
18	2,8903715755	....
19	2,9444387934	....
20	2,9957320845	....
21	3,0445222456	....
22	3,0910422583	....
23	3,1354940181	....
24	3,1780536298	....
25	3,2188756217	....
26	3,2580963324	....
27	3,295836658	....
28	3,3322042999	....
29	3,3672956175	....
30	3,401197167	....



H_n ***and*** $log\ n$ ***are diverging numerical series***

H_n ***is an Algebraic numbers series***

$$log\ n = log\ e^x = x \in \{\mathbb{R}, \mathbb{C}\}$$

$log\ n$ ***is a Transcendental numbers series, assuming*** $n \in \mathbf{A}$ ***id est Algebraic***

$log\ n$ ***is an Algebraic series, assuming*** $x \in \mathbf{A}$ ***, hence*** n ***is a Transcendental series***

Monograph near completion. It will published as soon as possil

APPENDIX:

from Geometric series to Mercator series :

$$\sum_{k=0}^{x} n^k = 1 + n + n^2 + ... + n^x = \frac{1-n^{x+1}}{1-n}$$

$$\frac{1}{1+n} = 1 - n + n^2 - n^3 + ... \quad | \quad |n| < 1 \quad , \quad x \to +\infty$$

$$\int_0^x \frac{dn}{1+n} = \int_0^x \frac{(1+n)'}{1+n} dn = \int_0^x (1 - n + n^2 - n^3 + ...)\, dn$$

$$\left[\ln|1+n|\right]_0^x = \int_0^x dn - \int_0^x n\, dn + \int_0^x n^2 dn - \int_0^x n^3\, dn + ...$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} \quad dx$$

$$\Psi x = D'\, log(\Gamma x) = \frac{\Gamma' x}{\Gamma x}$$

$$\gamma = H_{n-1} - \psi(n)$$

a proof of the divergence of the harmonic series

$$\left[1 > \frac{1}{2}\right] \quad +$$

$$+ \left[\left(\frac{1}{2}\right) = \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 1$$

$$+ \left[\left(\frac{1}{3} + \frac{1}{4}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 2$$

$$+ \left[\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 4$$

$$+ \left[\left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 8$$

$$+ \left[\left(\cdots\cdots\cdots \frac{1}{32}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 16$$

$$+ \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad + \qquad 2^{n-1} = 32$$

proof of the irrationality of e
if e = a/b then a(b-1)! should be an integer

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \sim 2.718281828459.....$$

$$e = \frac{a}{b} = \sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

$$\sum_{n=0}^{b} \frac{1}{n!} \gtrapprox 2 < 2.718282.....$$

$$a(b-1)! = b!\left[\sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}\right]$$

$$b!\sum_{n=0}^{b} \frac{1}{n!} < a(b-1)! < b!\sum_{n=0}^{b} \frac{1}{n!} + 1$$

$$\pi \sim 3.1415926536$$

$$n! = \prod_{i=1}^{n} i$$

$$(n+1)! = (n+1)n!$$

$$n \mid (n-1)! + 1$$

if $\frac{n(n+1)}{2}$ *is true, then if* $\frac{(n-1)n}{2}$ *is true, then* $\frac{(n+1)(n+2)}{2}$ *is true, hence all others*

$$\frac{n(n+1)}{2} = \frac{n(n-1)}{2} + n$$

$$(p-1)! \equiv -1 (\mathrm{mod}\ p)$$

$$n^{p-1} \equiv 1 (\mathrm{mod}\ p)$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \quad \sim 2.7182818284$$

$$\begin{bmatrix} n \\ k \end{bmatrix} = \frac{n!}{k!\ (n-k)!}$$

$$(x+y)^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} x^{n-k} y^k$$

$$\sum_{n=0}^{\infty} \frac{f^n x_0}{n!} (x - x_0)^n$$

$$\frac{fx - f\xi}{x - \xi} = f^1 \Xi \quad \textit{where} \; \Xi \in (\xi , x) \quad \textit{then}$$

$$fx = f\xi + f^1 \Xi (x - \xi) \quad \textit{a case of Taylor + Lagrange remainder}$$

The first-order Taylor polynomial is the linear approximation of the function

$$\Gamma z = \int_0^{+\infty} t^{z-1} e^{-t} \; dt \qquad | Re(z) > 0$$

$$\Gamma(n+1) = n!$$

$$\beta(x,y) = \int_0^1 t^{x-1} (1-t)^{y-1} \; dt \qquad | \; x,y \in c : Re(z) > 0$$

Stirling

$$\Re(z)^+ \; , \quad \Gamma(z+1) = \prod(z) = z!$$

$$ln\Gamma(z) = \left(z - \frac{1}{2}\right) \ln z - z + \frac{\ln 2\pi}{2} + 2 \int_0^{\infty} \frac{\arctan \frac{t}{z}}{e^{2\pi t} - 1}$$

Bernoulli numbers

$$f x = \frac{x}{e^x - 1} \qquad f' x = \frac{(1-x)e^x - 1}{\left(e^x - 1\right)^2} \qquad f'' x = \frac{e^x\left((x-2)e^x + x + 2\right)}{\left(e^x - 1\right)^3}$$

$$B_n = \lim_{x \to 0} f^n x$$

$$B_0 = \lim_{x \to 0} \frac{x}{e^x - 1} = 1$$

$$B_1 = \lim_{x \to 0} \frac{(1-x)e^x - 1}{\left(e^x - 1\right)^2} = \frac{1}{2}$$

$$B_2 = \lim_{x \to 0} \frac{e^x\left((x-2)e^x + x + 2\right)}{\left(e^x - 1\right)^3} = \frac{1}{6}$$

$$\Gamma(x+1) =$$

$$= \int_0^\infty e^{-t} t^x \, dt = \left[-e^{-t} t^x\right]_0^\infty + \int_0^\infty e^{-t} x t^{x-1} \, dt = 0 + x \int_0^\infty e^{-t} t^{x-1} \, dt =$$

$$= x\Gamma(x) \qquad , x \in N \backslash \{0\}$$

$$\forall x \in N \backslash 0$$

$$x\Gamma(x) = x(x-1)\Gamma(x-1) = x(x-1)(x-2)\Gamma(x-2) = \ldots = x!$$

$$\begin{cases} 1! = 1 \\ (n+1)! = (n+1)n! \end{cases}$$

$$\begin{cases} \Gamma(1) = 1 \\ \Gamma(n+1) = n\Gamma(n) = n! \end{cases}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-t} t^{-\frac{1}{2}} \, dt$$

assuming $t = z^2 \qquad dt = 2z dz$

$$= 2\int_0^\infty e^{-z^2} z^{-1} z \, dz = \sqrt{\pi}$$

(Gaussian integral)

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

$$\Gamma\left(n + 1 + \frac{1}{2}\right) = \left(n + \frac{1}{2}\right)! = \frac{(2n+1)!!}{2^{n+1}}\sqrt{\pi}$$

$n \in N^+$

$$\left(-n + \frac{1}{2}\right)! = \Gamma\left(\frac{1}{2} - n + 1\right) = (-1)^{n-1} \frac{2^{n-1}}{(2n-3)!!}\sqrt{\pi}$$

$$\sum_{n=1}^\infty \frac{1}{n} - \int_1^\infty \frac{1}{n} \, dn = \sum_{n=1}^\infty \frac{1}{n} - \ln n_{\to\infty} = \int_1^\infty \frac{1}{\lfloor x \rfloor} - \frac{1}{x} \sim 0.5772156649$$

$$\gamma = \sum_{k=1}^{n\to\infty} \frac{1}{k} - \ln n - o(1)$$

where $$ln\left(1+\frac{1}{n\to\infty}\right) \simeq o(1)$$

then, concerning the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \ln\left(\frac{n+1}{n}\right)\right)$$

assuming

$$x \in (0,1] \quad \to \quad \ln(1+x)$$

in the first-order Taylor polynomial:

$$\frac{x - ln(1+x)}{x-1-x} \quad = \quad ln(1+x) - x \quad = \quad f^1$$

we omit **dx** ***in the integrals***

$$ln(1+x) = f^1 + x \; = \; x + \int ln(1+x) \quad - \int x \; =$$

$$= x - \frac{x^2}{2} + \int ln(1+x) \; = \; x - \frac{x^2}{2} + \left[x\, ln(1+x) - \int \frac{x+1-1}{1+x}\right] =$$

$$= x - \frac{x^2}{2} + \left[x ln(1+x) - \int 1 \; + \int \frac{1}{1+x}\right] =$$

$$= x - \frac{x^2}{2} + x\, ln(1+x) - x \; + ln(1+x) \; = \; \frac{x^2}{2x} \; = \frac{x}{2}$$

$$0 \; < \; x - ln(1+x) \; < \; x$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \ln\left(\frac{n+1}{n}\right)\right)$$ ***this series converges by the comparison criterion***

$$\gamma = \sum_{n=1}^{\substack{n\to\infty \\ n-1}} \frac{1}{n} - \Psi(n)$$

$$\Psi(1) = \frac{e^{-t}}{-e^{-t}}$$ ***then*** $$-\gamma(1) = \gamma\Psi(1)$$

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y\left(x^2 - z^2\right)$$ ***Bessel f***

$$\infty \leftarrow \bar{9}9999 + \infty \leftarrow \bar{0}0001 = 0 \Longrightarrow \infty \leftarrow \bar{9}9999 = -1$$

to be continued ....