$n \to \pm \infty$ 

$$e^{e} = e^{\left(1+\frac{1}{n}\right)^{n}} = \sum_{m=0}^{\infty} \frac{\left(1+\frac{1}{n}\right)^{n^{m}}}{m!} = \sum_{j=0}^{m} {m \choose j} \frac{\left(1+\frac{1}{n}\right)^{n^{j}}}{m^{j}} =$$

$$= \sum_{j=0}^m \frac{\displaystyle\prod_{h=0}^{j-1} m-h}{j!} \cdot \frac{\left(1+\frac{1}{n}\right)^{nj}}{m^j} >$$

$$> \left(\frac{1}{0!}\right)^{\left(\frac{1}{0!}\right)} + \left(\frac{1}{1!}\right)^{\left(\frac{1}{1!}\right)} + \left(\frac{1}{3!}\right)^{\left(\frac{1}{3!}\right)} + \dots + \left(\frac{1}{n!}\right)^{\left(\frac{1}{n!}\right)}$$

$$e^{x} = \frac{\left(1 + \frac{x}{n}\right)^{n}}{\lim \to \infty} = \sum_{k=0}^{n \to \infty} \langle n \\ k \rangle \left(\frac{x}{n}\right)^{k} , x \in \mathbb{R}, \mathbb{C}$$

$$e^{iy},_{then} iy = \log \sum_{n=0}^{\infty} \frac{(iy)^n}{n!},_{yhere} y \in \mathbb{R}$$

$$\frac{1}{y} \log \sum_{n=0}^{\infty} \frac{(iy)^n}{n!} = \sqrt{-1}$$

 $(iy)^n$  is an alternating series absolutely convergent

$$n! > (iy)^{n}_{, then} \frac{(iy)^{n}}{n!} < 1$$

$$\sum_{n=0}^{\infty} \frac{(iy)^n}{n!} \simeq 0$$

$$\ln(\sim 0^+) = -\lambda_{, where} \lambda \in \mathbb{R}^+_{is very large}$$

$$-\frac{\lambda}{y}=i$$

$$-\,\lambda \,{=}\,\lambda \,{\in}\, R^{\scriptscriptstyle -}$$

$$\sqrt{\frac{\lambda}{y}} = -1 \rightarrow y \in \mathbb{R}^+$$

$$e^{iy} = cosy - \frac{\lambda}{y}siny = e^{\frac{\lambda}{y}y} = e^{\lambda}_{,BUT}\lambda \neq iy$$

$$q = x + yi + \xi j + vk$$

<sup>where</sup>  
x,y,
$$\xi, \upsilon \in \mathbb{R}$$
  
 $i^2 = j^2 = k^2 = ijk = -1$   
 $ij = k$ ,  $jk = i$ ,  $ki = j$ ,  $ji = -k$ ,  $kj = -i$ ,  $ik = -j$ 

$$e^{q} = e^{x} e^{yi} e^{\xi j} e^{\upsilon k} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} \cdot \sum_{n=0}^{\infty} \frac{(yi)^{n}}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\xi j)^{n}}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\upsilon k)^{n}}{n!}$$

$$= \left(1 + x + \frac{x^2}{2} + \dots\right) \left(1 + iy - \frac{y^2}{2} - \frac{iy^3}{6} + \frac{y^4}{24} + \frac{iy^5}{120} - \dots\right) (\dots) (\dots)$$

trigonometric function

$$e^{q} = \sum_{n=0}^{\infty} \frac{\widehat{x}^{n}}{n!} \begin{vmatrix} c(y)c(\xi)c(v) &+ jc(y)s(\xi)c(v) + \\ + is(y)c(\xi)c(v) &+ ks(y)s(\xi)c(v) + \\ + kc(y)c(\xi)s(v) &+ ic(y)s(\xi)s(v) - \\ -js(y)c(\xi)s(v) &- s(y)s(\xi)s(v) \end{vmatrix}$$

to be continued...

$$\left(\left\{e^{x}\right\} = e^{x} - \left\lfloor e^{x}\right
ight
floor
ight) \in \left[0,1
ight)$$

$$\ln n = \ln e^{x} = \ln \left[ \left[ e^{x} \right] + \left\{ e^{x} \right\} \right] = \ln \left[ \left[ e^{x} \right] \left[ \frac{\left[ e^{x} \right] + \left\{ e^{x} \right\} \right]}{\left[ e^{x} \right]} \right] \right]$$
$$= \ln \left[ e^{x} \right] + \ln \left[ 1 + \frac{\left\{ e^{x} \right\}}{\left[ e^{x} \right]} \right] = \ln \left[ e^{x} \right] + \sum_{n=1}^{+\infty} \frac{\left( -1 \right)^{n+1}}{n} \left[ \frac{\left\{ e^{x} \right\}}{\left[ e^{x} \right]} \right]^{n}$$

where

$$\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \left( \frac{\left\{ e^{x} \right\}}{\left\lfloor e^{x} \right\rfloor} \right)^{n} = log(1+\xi)$$

 $\xi = 1_{\text{generates a particular Mercator series like an alternating harmonic CONVERGING series!}$  (although not for absolut values).

but assuming

by reductio ad absurdum, the Mercator series is not defined by  $\log (n \in \mathbb{N})_{\text{for}} e^x \in \mathbb{N} \to \{\} = 0_{(\text{mantissa = zero})}$ 

proof of the irrationality of e if e = a/b then a(b-1)! should be an integer

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \sim 2.718281828459....$$

$$e = \frac{a}{b} = \sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

$$\sum_{n=0}^{b} \frac{1}{n!} \geq 2 < 2.718282....$$

$$a(b-1)! = b! \left[ \sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!} \right]$$

$$b! \sum_{n=0}^{b} \frac{1}{n!} < a(b-1)! < b! \sum_{n=0}^{b} \frac{1}{n!} + 1$$

from Geometric series to Mercator series :

$$\sum_{k=0}^{x} n^{k} = 1 + n + n^{2} + \dots + n^{x} = \frac{1 - n^{x+1}}{1 - n}$$

$$\frac{1}{1 + n} = 1 - n + n^{2} - n^{3} + \dots + |n| < 1 \quad , \quad x \to +\infty$$

$$\int_{0}^{x} \frac{dn}{1 + n} = \int_{0}^{x} \frac{(1 + n)^{1}}{1 + n} dn = \int_{0}^{x} (1 - n + n^{2} - n^{3} + \dots) dn$$

$$[\ln|1 + n|]_{0}^{x} = \int_{0}^{x} dn - \int_{0}^{x} n dn + \int_{0}^{x} n^{2} dn - \int_{0}^{x} n^{3} dn + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}$$

## a proof of the divergence of the harmonic series

•

$$\begin{bmatrix} 1 > \frac{1}{2} \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{1}{2}\right) = \frac{1}{2} \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{1}{3} + \frac{1}{4}\right) > \frac{1}{2} \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{1}{3} + \frac{1}{4}\right) > \frac{1}{2} \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > \frac{1}{2} \end{bmatrix} + \\ + \begin{bmatrix} \left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) > \frac{1}{2} \end{bmatrix} + \\ 2^{n-1} = 4 \\ + \begin{bmatrix} \left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) > \frac{1}{2} \end{bmatrix} + \\ 2^{n-1} = 8 \\ + \begin{bmatrix} \left(\dots + \frac{1}{32}\right) > \frac{1}{2} \end{bmatrix} + \\ 2^{n-1} = 16 \\ + \dots + \\ 2^{n-1} = 32 \end{bmatrix}$$