Disquisitions on exp(x)

$n \to \pm\infty$

$$e^e = e^{\left(1+\frac{1}{n}\right)^n} = \sum_{m=0}^{\infty} \frac{\left(1+\frac{1}{n}\right)^{n^m}}{m!} = \sum_{j=0}^{m} \begin{bmatrix} m \\ j \end{bmatrix} \frac{\left(1+\frac{1}{n}\right)^{n^j}}{m^j} =$$

$$= \sum_{j=0}^{m} \frac{\prod_{h=0}^{j-1} m-h}{j!} \cdot \frac{\left(1+\frac{1}{n}\right)^{nj}}{m^j} >$$

$$> \left(\frac{1}{0!}\right)^{\left(\frac{1}{0!}\right)} + \left(\frac{1}{1!}\right)^{\left(\frac{1}{1!}\right)} + \left(\frac{1}{3!}\right)^{\left(\frac{1}{3!}\right)} + \cdots + \left(\frac{1}{n!}\right)^{\left(\frac{1}{n!}\right)}$$

$$e^x = \frac{\left(1+\frac{x}{n}\right)^n}{\lim \to \infty} = \sum_{k=0}^{n\to\infty} \left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle \left(\frac{x}{n}\right)^k \qquad , x \in \mathbb{R},\mathbb{C}$$

$$e^{iy}\text{, then } iy = \log \sum_{n=0}^{\infty} \frac{(iy)^n}{n!}\text{, where } y \in \mathbb{R}$$

$$\frac{1}{y}\log\sum_{n=0}^{\infty}\frac{(iy)^n}{n!} = \sqrt{-1}$$

$(iy)^n$ **is an alternating series absolutely convergent**

$n! > (iy)^n$ **, then** $\frac{(iy)^n}{n!} < 1$

$$\sum_{n=0}^{\infty}\frac{(iy)^n}{n!} \simeq 0$$

$\ln(\sim 0^+) = -\lambda$ **, where** $\lambda \in \mathbb{R}^+$ **is very large**

$$-\frac{\lambda}{y} = i$$

$$-\lambda = \lambda \in R^-$$

$\sqrt{\frac{\lambda}{y}} = -1 \quad \rightarrow y \in R^+$

$e^{iy} = \mathit{cos}y - \frac{\lambda}{y}\mathit{sin}y = e^{\frac{\lambda}{y}y} = e^{\lambda}$ **, BUT** $\lambda \neq iy$

$$\mathfrak{z} = x + yi + \xi j + \upsilon k$$

where

$$x, y, \xi, \upsilon \in \mathbb{R}$$

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k,\ jk = i,\ ki = j,\ ji = -k,\ kj = -i,\ ik = -j$$

$$e^{\mathfrak{z}} = e^x e^{yi} e^{\xi j} e^{\upsilon k} = \sum_{n=0}^{\infty} \frac{x^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(yi)^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\xi j)^n}{n!} \cdot \sum_{n=0}^{\infty} \frac{(\upsilon k)^n}{n!}$$

$$= \left(1 + x + \frac{x^2}{2} + \ldots\right)\left(1 + iy - \frac{y^2}{2} - \frac{iy^3}{6} + \frac{y^4}{24} + \frac{iy^5}{120} - \ldots\right)(\ldots)(\ldots)$$

trigonometric function

$$e^{\mathfrak{z}} = \sum_{n=0}^{\infty} \frac{\hat{x}^n}{n!} \begin{bmatrix} c(y)c(\xi)c(\upsilon) & + \ jc(y)s(\xi)c(\upsilon) + \\ + is(y)c(\xi)c(\upsilon) & + \ ks(y)s(\xi)c(\upsilon) + \\ + kc(y)c(\xi)s(\upsilon) & + ic(y)s(\xi)s(\upsilon) - \\ -js(y)c(\xi)s(\upsilon) & -s(y)s(\xi)s(\upsilon) \end{bmatrix}$$

to be continued...

$$\left(\left\{e^x\right\} = e^x - \left\lfloor e^x \right\rfloor\right) \in [0,1)$$

$$\ln n = \ln e^x = \ln\left[\lfloor e^x \rfloor + \{e^x\}\right] = \ln\left[\lfloor e^x \rfloor\left(\frac{\lfloor e^x \rfloor + \{e^x\}}{\lfloor e^x \rfloor}\right)\right]$$

$$= \ln\lfloor e^x \rfloor + \ln\left(1 + \frac{\{e^x\}}{\lfloor e^x \rfloor}\right) = \ln\lfloor e^x \rfloor + \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n}\left(\frac{\{e^x\}}{\lfloor e^x \rfloor}\right)^n$$

where

$$\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n}\left(\frac{\{e^x\}}{\lfloor e^x \rfloor}\right)^n = log(1+\xi)$$

where normally $\xi = 1$ **generates a particular Mercator series like an alternating harmonic CONVERGING series! (although not for absolut values).**

but assuming

$$\int_1^2 \frac{1}{n} = [ln\ n]_1^2 = ln\ 2 = ln(1+e^x)$$

$$\xi = \frac{\{e^x\}}{\lfloor e^x \rfloor} = \frac{0}{1}$$ **sets the Mercator series to zero, but** $$\xi = 1 = \frac{\{e^x\}}{\lfloor e^x \rfloor} = 0 \neq 1$$

by reductio ad absurdum, the Mercator series is not defined by $\log(n \in \mathbb{N})$ **for**

$e^x \in \mathbb{N} \rightarrow \{\} = 0$ **(mantissa = zero)**

proof of the irrationality of e
if e = a/b then a(b-1)! should be an integer

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \sim 2.718281828459.....$$

$$e = \frac{a}{b} = \sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}$$

$$\sum_{n=0}^{b} \frac{1}{n!} \gtreqless 2 < 2.718282.....$$

$$a(b-1)! = b!\left[\sum_{n=0}^{b} \frac{1}{n!} + \sum_{n=b+1}^{\infty} \frac{1}{n!}\right]$$

$$b!\sum_{n=0}^{b} \frac{1}{n!} < a(b-1)! < b!\sum_{n=0}^{b} \frac{1}{n!} + 1$$

from Geometric series to Mercator series :

$$\sum_{k=0}^{x} n^k = 1 + n + n^2 + ... + n^x = \frac{1-n^{x+1}}{1-n}$$

$$\frac{1}{1+n} = 1 - n + n^2 - n^3 + ... \quad | \quad |n| < 1 \quad , \quad x \to +\infty$$

$$\int_0^x \frac{dn}{1+n} = \int_0^x \frac{(1+n)'}{1+n} dn = \int_0^x (1 - n + n^2 - n^3 + ...)\, dn$$

$$\left[\ln|1+n|\right]_0^x = \int_0^x dn - \int_0^x n\, dn + \int_0^x n^2 dn - \int_0^x n^3\, dn + ...$$

$$\ln(1+x) \;=\; x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} x^n}{n}$$

a proof of the divergence of the harmonic series

$$\left[1 > \frac{1}{2}\right] \quad +$$

$$+ \left[\left(\frac{1}{2}\right) = \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 1$$

$$+ \left[\left(\frac{1}{3} + \frac{1}{4}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 2$$

$$+ \left[\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 4$$

$$+ \left[\left(\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 8$$

$$+ \left[\left(\ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{1}{32}\right) > \frac{1}{2}\right] \quad + \qquad 2^{n-1} = 16$$

$$+ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad + \qquad 2^{n-1} = 32$$

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