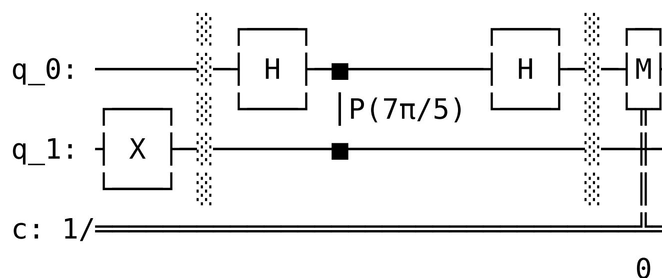
In [1]:

```
from qiskit.visualization import array_to_latex
from qiskit.quantum_info import Statevector
from qiskit.quantum_info.operators import Operator
import math

psi1 = Statevector([math.cos(math.pi / 8), math.sin(math.pi / 8)]
psi2 = Statevector([math.cos(5 * math.pi / 8), math.sin(5 * math

# When given a Statevector input, the Operator function returns t
# product of that state vector with itself — or, in other words,
# product of the vector times its conjugate transpose.

H = Operator(psi1) - Operator(psi2)

display(array_to_latex(H))
```

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

In [2]:

```
from math import pi, cos, sin
from qiskit import QuantumCircuit

theta = 0.7  # Can be changed to any value between 0 and 1

qc = QuantumCircuit(2, 1)

# Prepare eigenvector, which is the |1> state

qc.x(1)
qc.barrier()

# Implement the estimation procedure
qc.h(0)
qc.cp(2 * pi * theta, 0, 1)
qc.h(0)
qc.barrier()

# Perform the final measurement
qc.measure(0, 0)

# Draw the circuit
display(qc.draw())
```



In [3]:

```
from qiskit.primitives import Sampler

display(Sampler().run(qc).result().quasi_dists[0])
```

```
{0: 0.345491502812526, 1: 0.654508497187474}
```

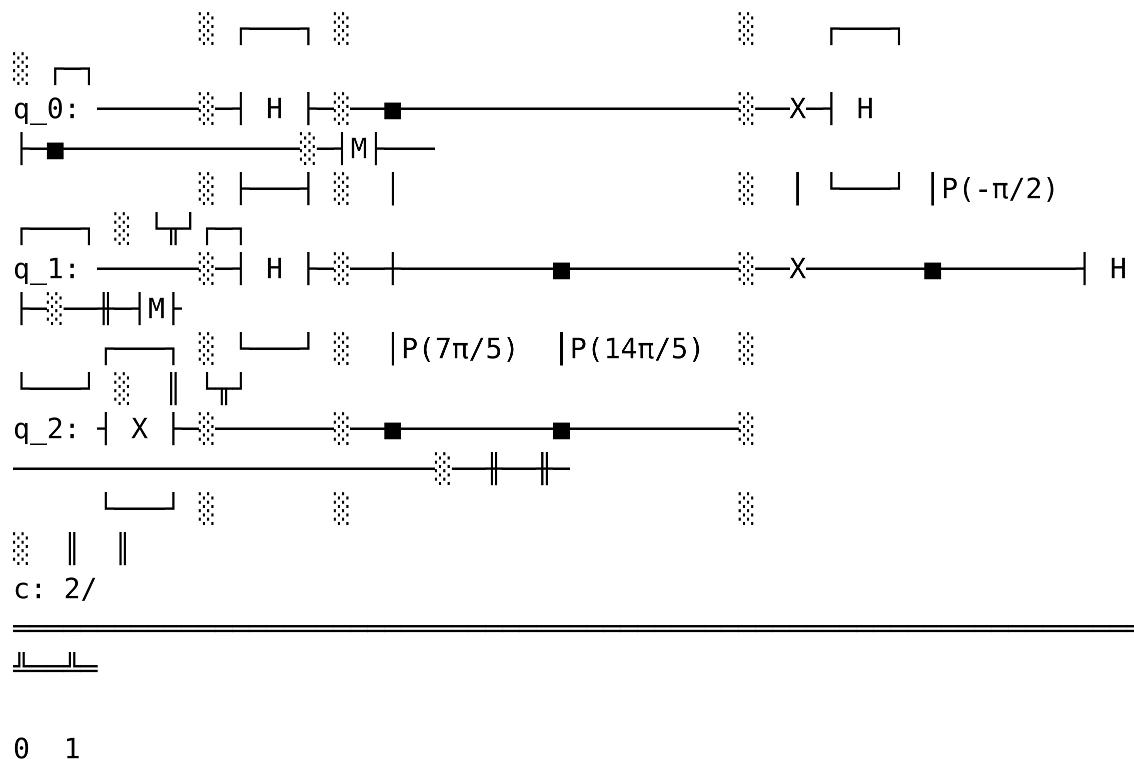
In [4]:

```
display({   # Calculate predicted results
    0: cos(pi * theta) ** 2,
    1: sin(pi * theta) ** 2
})
```

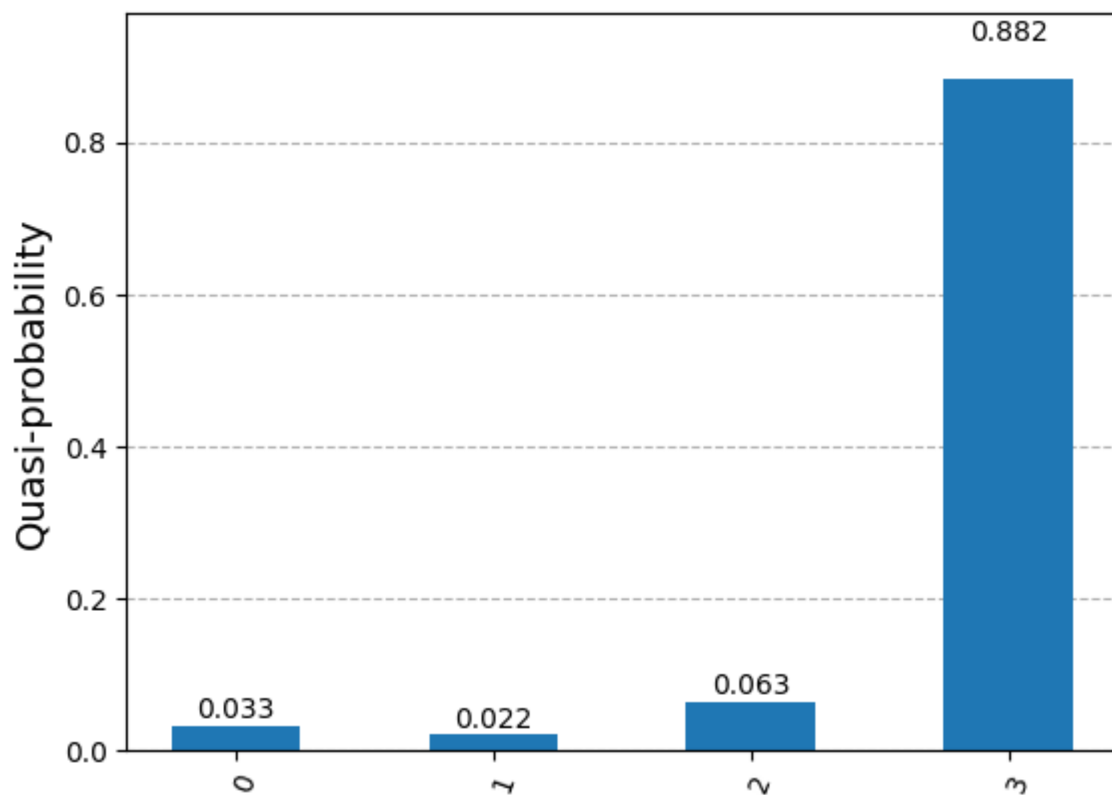
```
{0: 0.34549150281252616, 1: 0.6545084971874737}
```

In [5]:

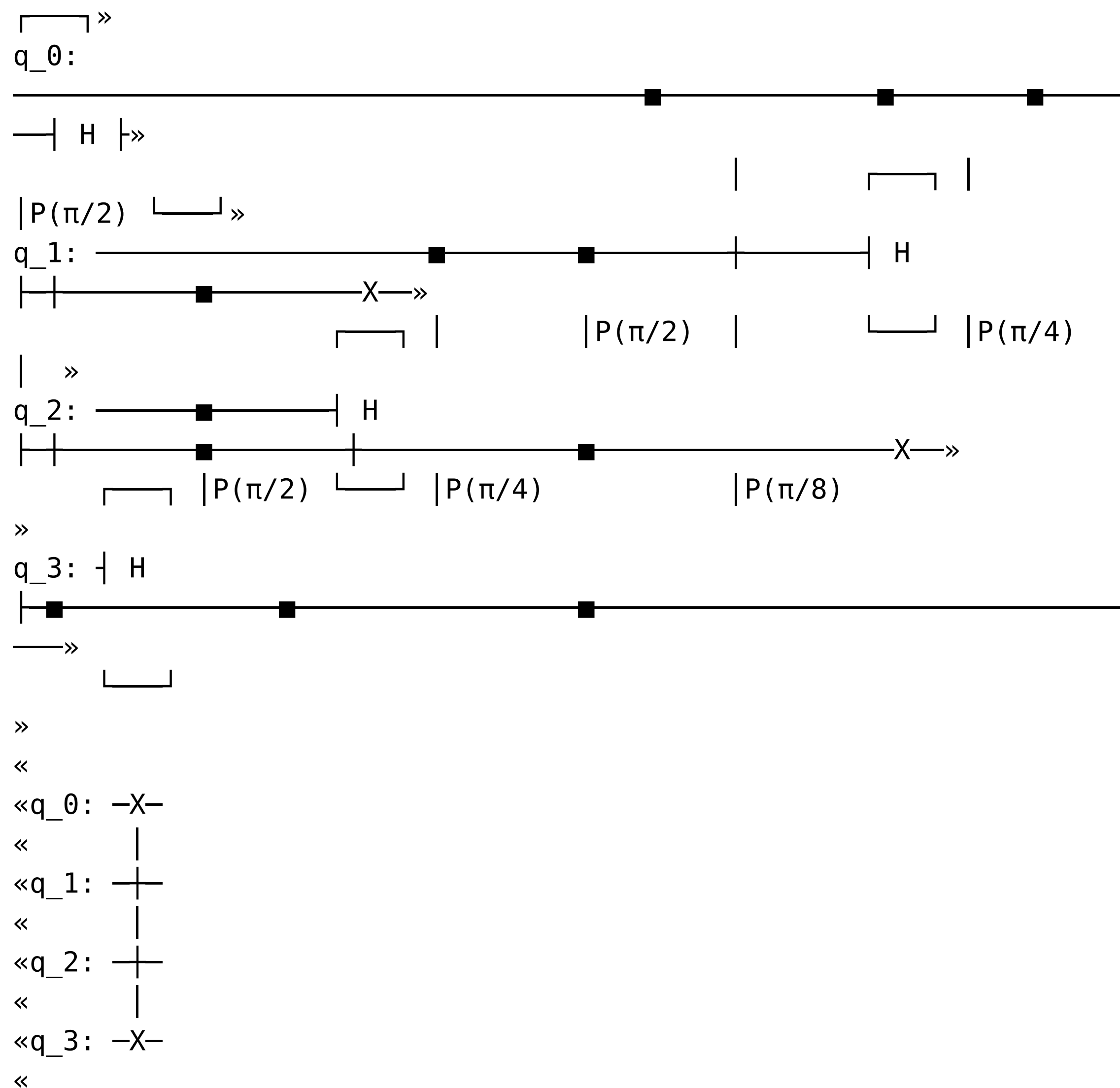
```
from math import pi
from qiskit import QuantumCircuit

theta = 0.7
qc = QuantumCircuit(3, 2)

# Prepare the eigenvector
qc.x(2)
qc.barrier()

# The initial Hadamard gates
qc.h(0)
qc.h(1)
qc.barrier()

# The controlled unitary gates
qc.cp(2 * pi * theta, 0, 2)
qc.cp(2 * pi * (2 * theta), 1, 2)
qc.barrier()

# An implementation of the inverse of the two-qubit QFT
qc.swap(0, 1)
qc.h(0)
qc.cp(-pi / 2, 0, 1)
qc.h(1)
qc.barrier()

# And finally the measurements
qc.measure([0, 1], [0, 1])
display(qc.draw())
```



In [6]:
```
from qiskit.visualization import plot_histogram

result = Sampler().run(qc).result()
display(plot_histogram(result.quasi_dists))
```

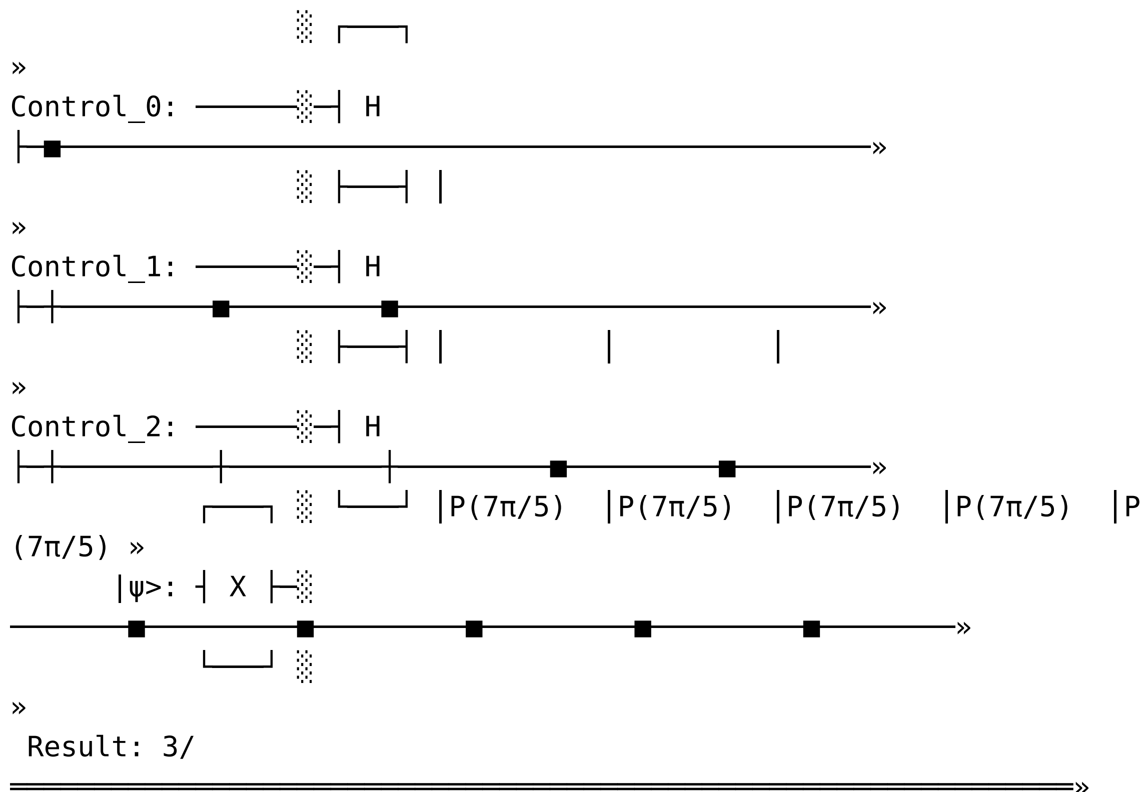


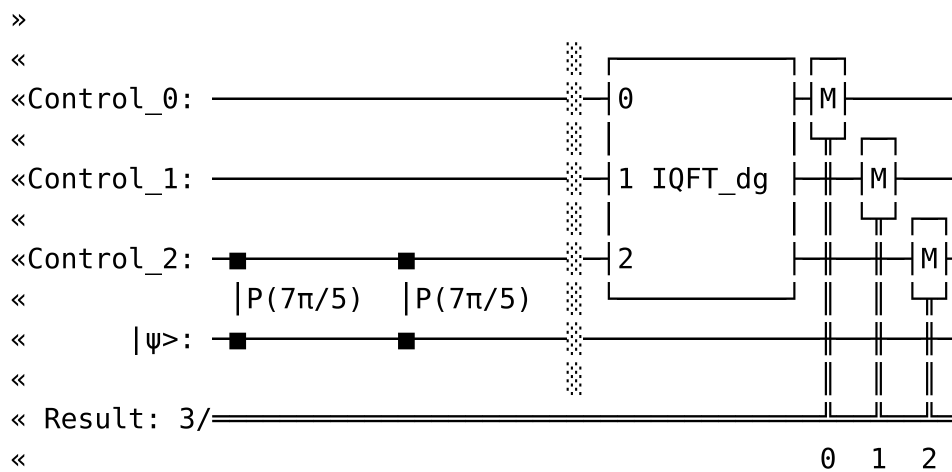
In [7]:
```
from qiskit.circuit.library import QFT

display(QFT(4).decompose().draw())
```



In [8]:

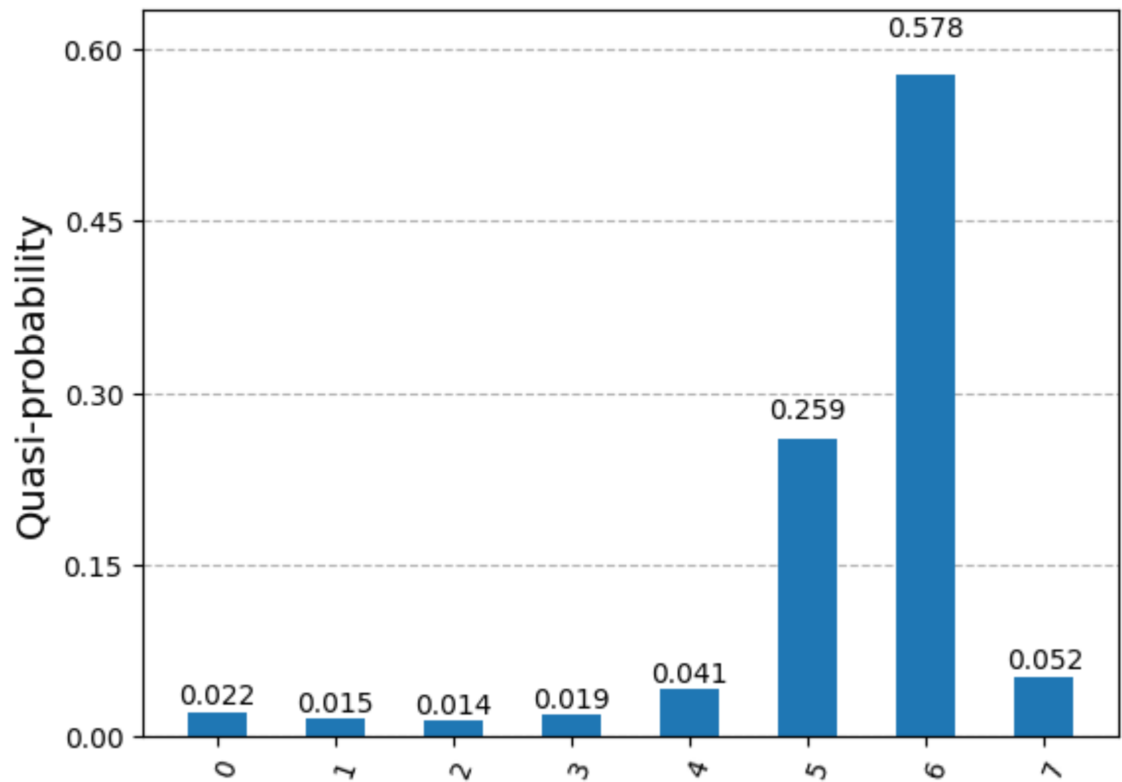
```
from qiskit import QuantumCircuit, QuantumRegister, ClassicalReg:
from qiskit.circuit.library import QFT

theta = 0.7
m = 3  # Number of control qubits

control_register = QuantumRegister(m, name="Control")
target_register = QuantumRegister(1, name="|ψ>")
output_register = ClassicalRegister(m, name="Result")
qc = QuantumCircuit(control_register, target_register, output_reg

# Prepare the eigenvector |ψ>
qc.x(target_register)
qc.barrier()

# Perform phase estimation
for index, qubit in enumerate(control_register):
    qc.h(qubit)
    for _ in range(2**index):
        qc.cp(2 * pi * theta, qubit, target_register)
qc.barrier()

# Do inverse quantum Fourier transform
qc.compose(
    QFT(m, inverse=True),
    inplace=True
)

# Measure everything
qc.measure(range(m), range(m))
display(qc.draw())
```


Control_0:
H
Control_1:
H
Control_2:
H
P(7π/5)
P(7π/5)
P(7π/5)
P(7π/5)
P(7π/5)
|ψ>:
X
Result: 3/


«Control_0:
«Control_1:
«Control_2:
P(7π/5)
P(7π/5)
« |ψ>:
« Result: 3/
0
1 IQFT_dg
2
M
M
M
0 1 2

In [9]:

```
1 result = Sampler().run(qc).result()
2 display(plot_histogram(result.quasi_dists))
```



In [10]:

```
1 most_probable = max(result.quasi_dists[0], key=result.quasi_dists
2 
3 print(f"Most probable output: {most_probable}")
4 print(f"Estimated theta: {most_probable/2**m}")
```

```
Most probable output: 6
Estimated theta: 0.75
```

In [11]:

```
def phase_estimation(
        controlled_operation: QuantumCircuit,
        psi_prep: QuantumCircuit,
        precision: int
    ):
    """
    Carry out phase estimation on a simulator.
    Args:
        controlled_operation: The operation to perform phase est:
                              controlled by one qubit.
        psi_prep: Circuit to prepare |ψ>
        precision: Number of counting qubits to use
    Returns:
        float: Best guess for phase of U|ψ>
    """
    control_register = QuantumRegister(precision)
    output_register = ClassicalRegister(precision)

    target_register = QuantumRegister(psi_prep.num_qubits)
    qc = QuantumCircuit(control_register, target_register, output

    # Prepare |ψ>
    qc.compose(psi_prep,
               qubits=target_register,
               inplace=True)

    # Do phase estimation
    for index, qubit in enumerate(control_register):
        qc.h(qubit)
        for _ in range(2**index):
            qc.compose(
                controlled_operation,
                qubits=[qubit] + list(target_register),
                inplace=True,
            )

    qc.compose(
        QFT(precision, inverse=True),
        qubits=control_register,
        inplace=True
    )

    qc.measure(control_register, output_register)

    measurement = Sampler().run(qc, shots=1).result().quasi_dists
    return measurement / 2**precision
```

In [12]:
```
psi_prep = QuantumCircuit(4)
psi_prep.x(0)
display(psi_prep.draw())
```

```
     ┌───┐
q_0: ┤ X ├
     └───┘
q_1: ─────

q_2: ─────

q_3: ─────
```

In [1]:
```
N = 21
a = 17
max_power = 12

print("k \t a^k \n")
for k in range(1, max_power + 1):
    print(
        "%2d \t %2d" % (k, a**k % N)
    )  # The % operation computes the remainder modulo N
```

```
k 	 a^k 

 1 	 17
 2 	 16
 3 	 20
 4 	  4
 5 	  5
 6 	  1
 7 	 17
 8 	 16
 9 	 20
10 	  4
11 	  5
12 	  1
```

In [2]:

```
N = 21
a = 18
max_power = 12

print("k \t a^k \n")
for k in range(1, max_power + 1):
    print("%2d \t %2d" % (k, a**k % N))
```

```
k        a^k

 1       18
 2        9
 3       15
 4       18
 5        9
 6       15
 7       18
 8        9
 9       15
10       18
11        9
12       15
```

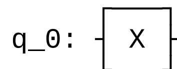
In [3]:

```
def c_amod15(a):
    """
    Controlled multiplication by a mod 15.
    This is hard-coded for simplicity.
    """
    if a not in [2, 4, 7, 8, 11, 13]:
        raise ValueError("'a' must not have common factors with 1
    U = QuantumCircuit(4)
    if a in [2, 13]:
        U.swap(2, 3)
        U.swap(1, 2)
        U.swap(0, 1)
    if a in [7, 8]:
        U.swap(0, 1)
        U.swap(1, 2)
        U.swap(2, 3)
    if a in [4, 11]:
        U.swap(1, 3)
        U.swap(0, 2)
    if a in [7, 11, 13]:
        for q in range(4):
            U.x(q)
    U = U.to_gate()
    U.name = f"{a} mod 15"
    c_U = U.control()
    return c_U
```

In [6]:

```
from qiskit import QuantumCircuit
def phase_estimation(
        controlled_operation: QuantumCircuit,
        psi_prep: QuantumCircuit,
        precision: int
    ):
    """
    Carry out phase estimation on a simulator.
    Args:
        controlled_operation: The operation to perform phase esti
                              controlled by one qubit.
        psi_prep: Circuit to prepare |ψ>
        precision: Number of counting qubits to use
    Returns:
        float: Best guess for phase of U|ψ>
    """
    control_register = QuantumRegister(precision)
    output_register = ClassicalRegister(precision)

    target_register = QuantumRegister(psi_prep.num_qubits)
    qc = QuantumCircuit(control_register, target_register, outpu

    # Prepare |ψ>
    qc.compose(psi_prep,
               qubits=target_register,
               inplace=True)

    # Do phase estimation
    for index, qubit in enumerate(control_register):
        qc.h(qubit)
        for _ in range(2**index):
            qc.compose(
                controlled_operation,
                qubits=[qubit] + list(target_register),
                inplace=True,
            )

    qc.compose(
        QFT(precision, inverse=True),
        qubits=control_register,
        inplace=True
    )

    qc.measure(control_register, output_register)

    measurement = Sampler().run(qc, shots=1).result().quasi_dist
    return measurement / 2**precision
```

In [7]:
```
psi_prep = QuantumCircuit(4)
psi_prep.x(0)
display(psi_prep.draw())
```



```
q_0: ─[ X ]─
q_1: ───────
q_2: ───────
q_3: ───────
```

In [21]:
```
from qiskit import QuantumCircuit, QuantumRegister, ClassicalReg
from qiskit.circuit.library import QFT
from qiskit.primitives import Sampler
from fractions import Fraction
from math import gcd

a = 8
N = 15

FACTOR_FOUND = False
ATTEMPT = 0
while not FACTOR_FOUND:
    ATTEMPT += 1
    print(f"\nAttempt {ATTEMPT}")
    phase = phase_estimation(
             c_amod15(a),
             psi_prep,
             precision=8
    )
    frac = Fraction(phase).limit_denominator(N)
    r = frac.denominator
    if phase != 0:
        # Guess for a factor is gcd(x^{r/2} - 1 , 15)
        guess = gcd(a ** (r // 2) - 1, N)
        if guess not in [1, N] and (N % guess) == 0:
            # Guess is a factor!
            print(f"Non-trivial factor found: {guess}")
            FACTOR_FOUND = True
```

```
Attempt 1
Non-trivial factor found: 3
```