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3D COMPLEX NUMBER

PREFACE

The fundamental theorem of algebra describes the advantages of the utilzation of the complex numbers and their conjugates.

Let another kind of complex number be as: $\boldsymbol{\zeta} = \mathbf{x} + \mathbf{iy} + \mathbf{iz}$, where $\mathbf{x}$, $\mathbf{z}$ = horizontal axes ; $\mathbf{y}$ = vertical axis, and where $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{R}$

$\boldsymbol{\zeta}$ is not a Quaternion or other Cayley-Dickson construction. $\mathbf{i}^3 \neq (\mathbf{ijk} = -\mathbf{1})$

we note that $|\boldsymbol{\zeta}| = \sqrt{\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^3 + 2yz} \neq \boldsymbol{\rho} = \sqrt{\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2}$

But, Let assume $\mathbf{s} = \mathbf{x} + \mathbf{iz}$, a complex number that replaces $\mathbf{x}$ in a 2dimensional complex plane.
Then we assume that the modulus $\mathbf{s}$ can rotate in the closed set $[0, 2\boldsymbol{\pi}]$. To use $\mathbf{s}$ instead of $\mathbf{x}$ allows us to use three-dimensional space while maintaining the features of complex numbers. So, $\mathbf{s}$ that intrinsically includes $\mathbf{i}$ except the cases where the interaxles angle is $\mathbf{n}$ $\boldsymbol{\pi}$, $\mathbf{n}$ = integer, is a dependent variable and it is mobile in all $\mathbf{x}$ + $\mathbf{z}$ plane , by assigning appropriate values at $\mathbf{x}$ and $\mathbf{z}$. The position of $\mathbf{s}$ can range in $[0, 2\boldsymbol{\pi}]$ by the infinite factors of $\boldsymbol{\pi}$

Then

$$\mathfrak{z} = s + iy$$

$$\mathbf{r} = \sqrt{\mathbf{x}^2 + \mathbf{z}^2}$$

$$|\mathfrak{z}| = \sqrt{\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2} = \boldsymbol{\rho} \in \mathbf{R}$$

Moreover if

$$\mathbf{f}(\boldsymbol{\zeta}) \equiv \mathbf{f}(\mathbf{x} + \mathbf{iy} + \mathbf{iz}) = \mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{z}) + \mathbf{iv}(\mathbf{x},\mathbf{y},\mathbf{z}) + \mathbf{iw}(\mathbf{x},\mathbf{y},\mathbf{z})$$

$$\mathbf{f}'(\boldsymbol{\zeta}_0) = {}_{\Delta\zeta\to 0}\frac{\mathbf{f}(\boldsymbol{\zeta}_0 + \Delta\boldsymbol{\zeta}) - \mathbf{f}(\boldsymbol{\zeta}_0)}{\Delta\boldsymbol{\zeta}}$$

$$\Delta\boldsymbol{\zeta} = (\Delta\mathbf{x}, \Delta\mathbf{y}, \Delta\mathbf{z}) , \quad \Delta\boldsymbol{\zeta}_\mathbf{x} = (\Delta\mathbf{x}, 0, 0) , \quad \Delta\boldsymbol{\zeta}_\mathbf{y} = (0, \Delta\mathbf{y}, 0) , \quad \Delta\boldsymbol{\zeta}_\mathbf{z} = (0, 0, \Delta\mathbf{z})$$

$$\mathbf{u}_\mathbf{x} = \mathbf{u}(\mathbf{x}_0 + \Delta\mathbf{x}, \mathbf{y}_0, \mathbf{z}_0) - \mathbf{u}(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$$

$$\mathbf{v}_\mathbf{x} = \mathbf{v}(\mathbf{x}_0 + \Delta\mathbf{x}, \mathbf{y}_0, \mathbf{z}_0) - \mathbf{v}(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$$

$$\mathbf{w}_\mathbf{x} = \mathbf{w}(\mathbf{x}_0 + \Delta\mathbf{x}, \mathbf{y}_0, \mathbf{z}_0) - \mathbf{w}(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$$

$$\mathbf{f}'(\boldsymbol{\zeta}_{x_0}) = {}_{\Delta\zeta_\mathrm{x}\to 0}\frac{\mathbf{u}_\mathbf{x} + \mathbf{iv}_\mathbf{x} + \mathbf{iw}_\mathbf{x}}{\Delta\boldsymbol{\zeta}_\mathbf{x}}$$

also by $\mathbf{u}_\mathbf{y}, \mathbf{v}_\mathbf{y}, \mathbf{w}_\mathbf{y}$ we have $\mathbf{f}'(\boldsymbol{\zeta}_{y_0}) = {}_{\Delta\zeta_y\to 0}\frac{\mathbf{u}_\mathbf{y} + \mathbf{iv}_\mathbf{y} + \mathbf{iw}_\mathbf{y}}{\mathbf{i}\Delta\boldsymbol{\zeta}_y} = \mathbf{f}'(\boldsymbol{\zeta}_{y_0}) = {}_{\Delta\zeta_y\to 0}\frac{-\mathbf{iu}_\mathbf{y} + \mathbf{v}_\mathbf{y} + \mathbf{w}_\mathbf{y}}{\Delta\boldsymbol{\zeta}_y}$

also by $\mathbf{u_z}, \mathbf{v_z}, \mathbf{w_z}$ we have $\mathbf{f'(\zeta_{z_0})} = {}_{\Delta\zeta_z \to 0} \frac{\mathbf{u_z + iv_z + iw_z}}{\mathbf{i\Delta\zeta_z}} = \mathbf{f'(\zeta_{z_0})} = {}_{\Delta\zeta_z \to 0} \frac{\mathbf{-iu_z + v_z + w_z}}{\mathbf{\Delta\zeta_z}}$

As $\mathbf{f(\zeta_0) = a + ib + ic}$ hence we have

$$\mathbf{a} = \lim_{\Delta\zeta_x \to 0} \frac{u_x}{\Delta\zeta_x}, \quad \lim_{\Delta\zeta_y \to 0} \frac{v_y}{\Delta\zeta_y}, \quad \lim_{\Delta\zeta_y \to 0} \frac{w_y}{\Delta\zeta_y}, \quad \lim_{\Delta\zeta_z \to 0} \frac{v_z}{\Delta\zeta_z}, \quad \lim_{\Delta\zeta_z \to 0} \frac{w_z}{\Delta\zeta_z}$$

$$\mathbf{b} = \lim_{\Delta\zeta_x \to 0} \frac{v_x}{\Delta\zeta_x}, \quad \lim_{\Delta\zeta_y \to 0} -\frac{u_y}{\Delta\zeta_y}$$

$$\mathbf{c} = \lim_{\Delta\zeta_x \to 0} \frac{w_x}{\Delta\zeta_x}, \quad \lim_{\Delta\zeta_z \to 0} -\frac{u_z}{\Delta\zeta_z}$$

Holomorphy Conditions

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} = \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x} \end{cases}$$

and deriving partially by $\mathbf{x}$ on $\mathbf{a}$, $\mathbf{y}$ on $\mathbf{b}$, $\mathbf{z}$ on $\mathbf{c}$, we obtain $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial v}{\partial x \partial z} + \frac{\partial w}{\partial x \partial y}$

But if

in $\mathfrak{z}$ the Holomorphy conditions are the same in 2d-complex number:

$$\begin{cases} \frac{\partial \sigma}{\partial s} = \frac{\partial v}{\partial y} \\ \frac{\partial \sigma}{\partial y} = -\frac{\partial v}{\partial s} \end{cases}$$

the Cauchy-Riemann equations in $\mathfrak{z} = \mathbf{x + iz}$ are $\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x} \end{cases}$,

deriving partially the first equation by $\mathbf{x}$ and the second equation by $\mathbf{z}$ then deriving partially the first equation by $\mathbf{z}$ and the second equation by $\mathbf{x}$, we obtain $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 \sigma}{\partial s^2} = 0$

and about $\begin{cases} \frac{\partial \sigma}{\partial s} = \frac{\partial v}{\partial y} \\ \frac{\partial \sigma}{\partial y} = -\frac{\partial v}{\partial s} \end{cases}$ deriving partially the first equation by $\mathbf{s}$ and the second equation by $\mathbf{y}$, we obtain

$$\frac{\partial^2 \sigma}{\partial s^2} + \frac{\partial^2 \sigma}{\partial y^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 \sigma}{\partial y^2} = 0$$

,

instead, deriving partially the first equation by $\mathbf{y}$ and the second equation by $\mathbf{s}$, we obtain $\frac{\partial^2 v}{\partial s^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$

So we obtain the Laplacian on 3D on ***v-vector*** but **not** on ***u-vector*** neither ***w-vector*** . The laplacian equation is only on direction of axis $\mathbf{y}$ that links the north pole and south pole of the sphere $\sum_{0}^{n}\sum_{1}^{n}\mathfrak{z}$ obtained by the Normed space consisting of vectors $\mathfrak{z}_n$ and $v(x,y,z)$ is the harmonic function of the laplacian equation on the ***v-vector***

Trigonometric coordinates and Eulerian equations:

$$s = r(\cos\theta + i\sin\theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$i\sin\phi = i\frac{e^{i\phi} - e^{-i\phi}}{2i}$$

$$\theta = \text{horizontal angle} \ ; \quad \phi = \text{vertical angle}$$

then $$\cos\theta + i\sin\theta + i\sin\phi = \frac{2e^{i\theta} + e^{i\phi} - e^{-i\phi}}{2}$$

$$r = \sqrt{x^2 + z^2} \ , \ \rho = \sqrt{s^2 + y^2}$$

then $$\mathfrak{z} = \frac{\rho}{2}\left(2re^{i\theta} + e^{i\phi} - e^{-i\phi}\right)$$

If $\theta = \pi \quad \phi = \pi$, then $\mathfrak{z} = \frac{\rho}{2}\left(2re^{i\theta} + e^{i\phi} - e^{-i\phi}\right) = -\rho r$ by Euler's identity

then also by De Moivre equations , with n = integer

$$\theta, \phi = (2n+1)\pi \longrightarrow \mathfrak{z} = -\rho \cdot r = -1$$, (dot product)

$$\theta, \phi = 2n\pi \longrightarrow \mathfrak{z} = \rho \cdot r = 1$$, (dot product)

Since $\theta, \phi \in [0, n\pi]$

PI/4 approximately is 3.14159265358979/4 = 0.785398163397448

$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = 0,707106781186548$$

$i = \pm\sqrt{-1}$, considering $e^{-i\phi} = \cos\phi - i\sin\phi$ hence $e^{i\phi} - e^{-i\phi} = 2i\sin\phi$

$$\mathfrak{z} \in \cong [-1,414213562373096\rho r - 0,707106781186548 \quad ; \quad 1,414213562373096\rho r + 0,707106781186548]$$

And if $\theta = j\pi \quad \phi = k\pi$ where $j, k \in \mathbb{R}$

$$\mathfrak{z} = \frac{\rho}{2}(2r(\cos j\pi + i\sin j\pi) + 2i\sin k\pi)$$

How wrong are we in calculating the position of the point $\mathfrak{z}$ and its vector? we need to neutralize, as far as possible, the irrationality of π to reduce the margin of error.

eg: the angle have value π^e , then assuming π^e is not a transcendental number and not even an irrational, then, the calculation of the value of the angle would be more exact. in this case, the argument to be used for the calculation of the angles should be $\frac{n}{k}\pi^e$, $n,k \in Q$ instead of $\frac{n}{k}\pi$.

So in other case which π could be non-irrational, eg $j\pi + ke$, $\pi^{\sqrt{2}}$, the identification of the point position $\mathfrak{z}$ will be more precise.